Homework 10

PHYS798C Spring 2024
Due Thursday, 25 April, 2024

1 rf SQUID Properties Part I

Consider a single lumped Josephson junction that is connected by a superconducting loop of geometrical inductance L (otherwise known as a radio frequency superconducting quantum interference device, or rf SQUID for short).

(a) If there are no applied magnetic fields, show that the current in the loop can take on values given by

$$I = I_c \sin\left(\frac{2\pi LI}{\Phi_0}\right).$$

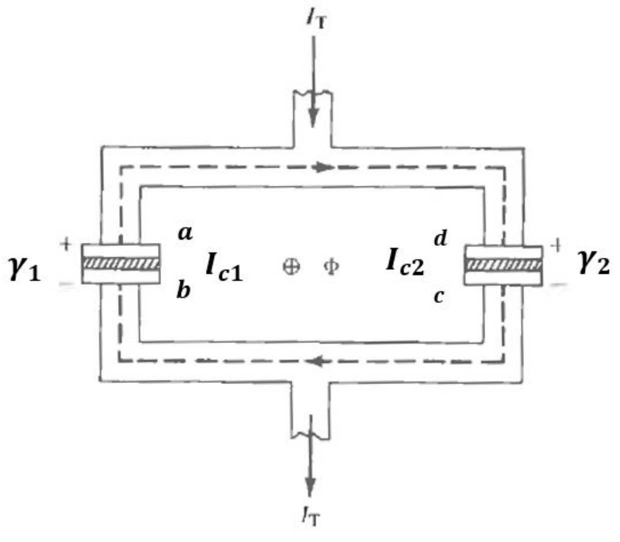
Find the allowed values of I/I_c for $LI_c = 6\Phi_0$. (One can define $\beta_{rf} \equiv 2\pi LI_c/\Phi_0$, in which case we are considering an rf SQUID with $\beta_{rf} = 12\pi$.)

(b) Show that the total energy of the rf SQUID carrying current I is given by

$$W = W_{Jos} - \frac{\Phi_0 I_c}{2\pi}\cos\left(\frac{2\pi LI}{\Phi_0}\right) + \frac{1}{2}LI^2, \text{ where } W_{Jos} = \Phi_0 I_c/2\pi.$$

Plot $(W - W_{Jos})/(\Phi_0 I_c)$ versus I/I_c for $LI_c = 6\Phi_0$. Show that all the allowed values except $I = 0$ are metastable, that is, only $I = 0$ is the true minimum.
Hint: To find the energy of the junction, integrate the power (iv) over time and convert the integral to one over the gauge-invariant phase γ.

2 Two JJs in Parallel: dc SQUID



Consider a symmetric parallel connection of two Josephson junctions in a DC SQUID geometry, as shown in the Figure. In the absence of fields and currents, the upper branch of the loop has a uniform superconducting order parameter, as well as the lower branch. The two branches are weakly coupled to each other through the two junctions. The gauge-invariant phase differences across the junctions are the same in the absence of field and current. The two Josephson junctions have critical currents I_{c1} and I_{c2}.

(a) If a magnetic field is applied perpendicular to the loop, show that $\gamma_2 = \gamma_1 - (2\pi\Phi/\Phi_0)$, (modulo 2π) where Φ is the flux in the loop. Assume that the two junctions are small enough such that the applied field has no effect on their individual critical currents, and that the currents circulating around the loop make no contribution to the flux in the loop, so that the total flux $\Phi = \Phi_{external}$. *Hint:* Assume that the fields and currents are screened to zero along the dashed line shown in the figure.

(b) With this fixed magnetic field, now add a transport current I_T to the SQUID, as shown in the Figure. Find the current through each junction.

(c) Assume that $I_{c1} = I_{c2} = I_c$ and find the maximum zero-voltage current that the SQUID loop can carry as a function of $\Phi = \Phi_{external}$, in terms of I_c and Φ_0 only.